Nuclear Physics
Final exam
Date: Wednesday, July 1, 2009
This exam has a total of 100 points

Problem #1. *(6 points)*
Estimate the approximate density of nuclear matter in g/cm^3. What would be the mass of a neutron star that had the diameter of an orange? Note that 1 atomic mass unit is 1.66×10^{-27} kg.

Problem #2. *(12 points)*

a) Write down the Bethe-Weizsäcker empirical mass formula (you don't need to know the constants. Here, they are given for your information: $a_V = 15.85$ MeV, $a_S = 18.34$ MeV, $a_a = 23.21$ MeV, $a_c = 0.71$ MeV and $a_p = 12$ MeV). *(4 points)*

c) Calculate the relation between the mass number A and the charge number Z for the most stable nuclei assuming large A (line of stability). *(4 points)*

d) Show for what value of Z^2/A the symmetric fission can take place. Ignore the pairing term in your calculations. Why is this not a very good approximation? *(4 points)*

Problem #3. *(7 points)*

a) Show that for elastic scattering of a high-energy electron with energy E_0 and momentum $kc \approx E_0$ with a heavy nucleus, the magnitude of the momentum transfered to the nucleus is $q = 2k\sin(\theta/2)$ where θ is the scattering angle. (4 points)

b) Is it justified to assume that $E_0 \approx kc$ for an electron kinetic energy of 20 MeV? Elaborate on your arguments. (3 points)

Problem #4. *(6 points)*

a) What is the energy dependence of the beta decay spectrum in a Kurie or Fermi plot for massless neutrinos. The Kurie plot is $(\frac{d\omega}{p^2dp})^{1/2}$ as a function of the electron energy. *(4 points)*

b) What happens in this plot when the neutrino is massive? Show this on a plot (Do NOT derive or write the explicit form of the formulas). *(2 points)*

Problem #5. *(10 points)*

a) The low lying levels of ${}^{13}_{6}$C are given by $\frac{1}{2}^{-}$ (0 MeV, ground state); $\frac{1}{2}^{+}$ (3.09 MeV), $\frac{3}{2}^{-}$ (3.68 MeV) $\frac{5}{2}^{+}$ (3.85 MeV). Justify these assignments. Does one need to change the order of some levels? *(5 points)*

b) Predict the first two excited states of ${}^{41}_{20}$Ca according to the extreme single-particle picture. *(5 points)*

Problem #6. *(10 points)*

The nucleus ^{176}Hf has ground and excited rotational state energy levels of: 0^{+} (0.0 MeV), 2^{+} (0.088 MeV), 4^{+} (0.290) MeV. Note that these are the energies of the leves.

a) Deduce the moment of inertia for this nucleus (in units of MeV^{-1}). Compare this value with the moment of inertia assuming the nucleus to be rigid rotor. The moment of inertia of rigid rotor with mass M and radius R is given by $\frac{2}{5}MR^2$.
(4 points)

b) What would you predict for the energy of the 6^{+} state? *(3 points)*

c) When the spin of the nucleus is very large the moment of inertia increases as a function of spin. Explain the effect. *(3 points)*

Problem #7. *(14 points)*

a) In a transition involving photons, give the parity selection rule, i.e. the parity change for a given electric and magnetic multipole ML and EL when going from the initial to final state. *(4 points)*

b) Why can we not have an $M1$ or $E1$ transition in nuclei in which the initial and final spins are both equal to 0? *(3 points)*

c) Why do nuclei and particles not have a permanent electric dipole moment? Elaborate on your answer. (3 points)

d) List all the possible multipolarities for the following $\gamma-$ray transitions, indicating in each case which transition(s) will be the most intense: (i) $3^{-} \rightarrow 2^{-}$; (ii) $\frac{5}{2}^{+} \rightarrow \frac{9}{2}^{+}$; (iii) $\frac{1}{2}^{+} \rightarrow \frac{1}{2}^{-}$; (iv) $\frac{3}{2}^{+} \rightarrow \frac{7}{2}^{-}$. *(4 points)*

Problem #8. *(8 points)*
Starting with the operator defined by

$$M(E\lambda,\mu) = \frac{3}{4\pi} ZeR_0^{\lambda}(\frac{\hbar}{\omega_\lambda B_\lambda})^{1/2}(b_{\lambda\mu} + (-1)^{\mu} b^{\dagger}_{\lambda-\mu}),$$

and

$$B(E\lambda, J_i \to J_f) = (2J_i+1)^{-1} \sum_{M_i M_f} | < J_f M_f | O_{\lambda\mu} | J_i M_i > |^2.$$

show that the following is correct:

$$B(E\lambda; \lambda \to 0^+) = (\frac{3R_0^{\lambda} Ze}{4\pi})^2 \frac{\hbar}{2\omega_\lambda B_\lambda}.$$

Problem #9. *(16 points)*
The nucleon-nucleon force is usually described as: (a) strong, (b) short-range, (c) charge symmetric, (d) charge independent, (e) containing both 'ordinary' and 'exchange' terms, (f) spin dependent, (g) non-central, and (h) saturating in nuclei. Explain the meaning of these statements and discuss briefly the experimental evidence for them.

Problem #10. *(11 points)*
The basic form of the Yukawa potential can be understood by considering the exchange of a spin-0 boson with mass m, obeying the static Kelin-Gordon equation:

$$(\nabla^2 - m^2c^2/\hbar^2)\Phi(r) = 0$$

a) Show that $\Phi(r) = V_0 \cdot \exp(-r/R)/(r/R)$ is a good solution (ignore the angular part of the equation and replace ∇^2 by $\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$). *(6 points)*

b) What is R? Give a physical interpretation for it and obtain it from the uncertainty principal. *(3 points)*

c) Why should the exchange particle for the strong force be a boson? *(2 points)*

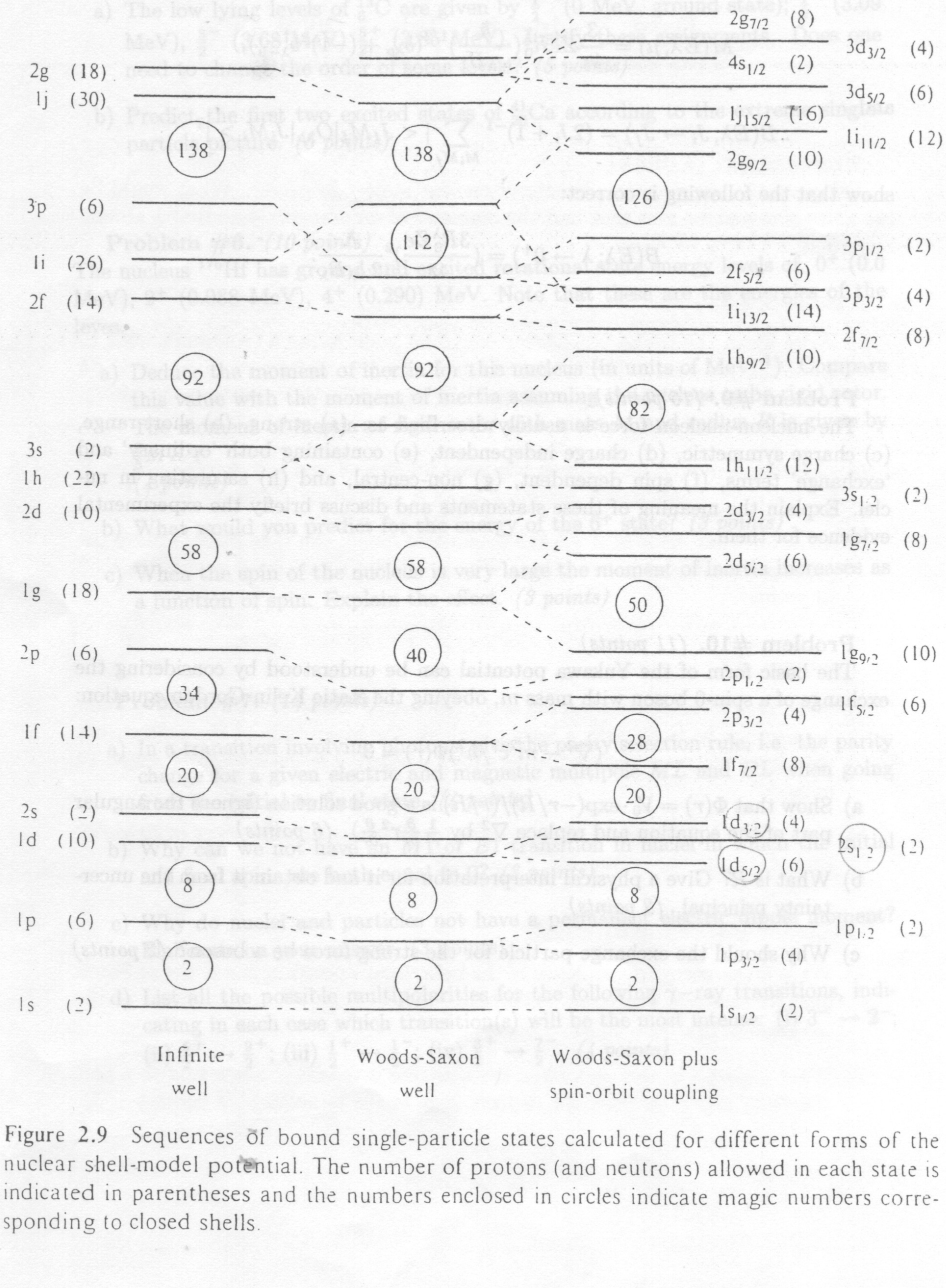

Figure 2.9 Sequences of bound single-particle states calculated for different forms of the nuclear shell-model potential. The number of protons (and neutrons) allowed in each state is indicated in parentheses and the numbers enclosed in circles indicate magic numbers corresponding to closed shells.